RESCUER

WORKSHOP 1

Schedule for day 1 (02/12/2024):
9:00 Opening, info about RESCUER (Henrik Rosa Maria)
9:30 The water-wave problem (Henrik)
10:30 Break, coffee
11:00 Introduction to Scientific Computing: ODEs and time-stepping methods (Magnus)
12:30 Lunch
13:30 Stability regions and Runge-Kutta methods (Magnus)
14:30 Break
15:00 Coding

Assignment for day 1.

The program **eu.m** *can be downloaded here:*
https://drive.google.com/drive/folders/1Om2D-dyGDOHWyZiqz3aY0GjmPtk4LDow

1) Consider the equation

$$\frac{dy}{dt} = -2y,$$

with initial value $y(0) = 1$. The exact solution of this equation is $y(t) = e^{-2t}$. For this problem you may use the program **eu.m** which can be found on the course website.

a) (Accuracy) Define the error E_N between the approximate solution and the exact solution by

$$E_N = \max_{j=0..N} |y(t_j) - y_N(t_j)|.$$

Use Euler's method and the fourth-order Runge-Kutta method to approximate a solution of the differential equation on the time interval $[0, 5]$. Using the commands *tic* and *toc*, determine which of the two methods is more efficient in obtaining an error E_N less than 10^{-2}, 10^{-4} and 10^{-6}.

b) (Stability) If the step size h is too large, then Euler's method produces an approximate solution that oscillates wildly or even grows exponentially. For the following four methods, experiment until you find the smallest value h_0, for which the approximation y_N describes the solution correctly, at least in a qualitative way. (This means that the computation does not collapse due to instability.)

- Euler's method
- Backwards Euler's method
- Trapezoidal method
- Fourth-order Runge-Kutta method

Schedule for day 2 (03/12/2024):
9:00 Coastal phase-resolved wave modeling (Volker)
10:30 Break
11:00 Well-posedness (Henrik)
12:00 Lunch
13:00 Finite differences: Periodic problems, method of lines, CFL (Magnus)
14:00 Break
14:30 Coding

Assignment for day 2.

Consider the periodic problem

$$\begin{aligned} u_t + 2u_x &= 0, \quad 0 \le x < 1, \quad t > 0, \\ u(x,0) &= \sin(2\pi x), \\ u(0,t) &= u(1,t). \end{aligned} \tag{1}$$

Discretise the x-axis using N equidistant intervals.

- Define the central second-order difference matrix approximating the spatial derivative and the boundary conditions.
- Compute its spectrum. How does this impact the choice of time integration scheme?
- Using the difference matrix, write down the semi-discrete approximation scheme.
- Use the RK4 routine from Day 1 to march the resulting ODE system in time.
- Calculate the analytical solution and use it to demonstrate that the code is design order accurate (at $t = 1$).

Schedule for day 3 (04/12/2024):
9:00 More about the water-wave problem (Henrik)
9:30 Understanding water waves with linear wave theory (Jupyter notebooks, Volker)
10:15 Break, coffee
10:45 Finite differences: Boundary conditions, SBP-SAT, injection (Magnus)
12:00 Lunch
13:00 Coding
14:00 Break, coffee
14:30 Afternoon free

Assignment for day 3.

Consider the initial-boundary value problem

$$\begin{aligned} u_t + 2u_x &= 0, \quad 0 < x < 1, \quad t > 0, \\ u(x,0) &= \sin(2\pi x), \\ u(0,t) &= \sin(-4\pi t). \end{aligned} \tag{2}$$

Discretise the x-axis using N equidistant intervals.

- Using *Diag4.m*, define 4th-order matrix operator that approximates the spatial derivative and the boundary condition.
- Compute its spectrum. How does the stable implementation of the boundary condition affect the spectrum in comparison with the periodic case?
- Use the RK4 routine from Day 1 to march the resulting ODE system in time.
- Calculate the analytical solution and use it to demonstrate that the code is design order accurate (at $t = 1$).

For the following system:

$$u_t + Au_x = 0, \quad 0 < x < 1, \quad t > 0, \tag{3}$$

$$A = \begin{pmatrix} -1 & -3 \\ -3 & -1 \end{pmatrix}, \tag{4}$$

- Compute the eigenvalues of A and determine the characteristic boundary conditions.
- Modify the previous code and use constant initial data. Demonstrate that a wave can introduced at the left boundary, pass the domain and exit on the right boundary.

Schedule for day 4 (05/12/2024):
9:00 Surf zone circulation (Andreas)
9:15 Nwogu equations (Anders)
9:30 Boussinesq theory, continued (Volker)
10:15 Break
10:45 Existence of solutions (Henrik)
11:15 Burgers equation: linearised stability (Magnus)
12:00 Lunch
13:00 SBP artificial diffusion. Burgers equation: scaling the diffusion, Roe, global and local Lax-Friedrichs (Magnus)
14:00 Coding
14:30 Break, coffee
15:00 Coding
16:00 Break
16:15 Presentation of career goals
17:00 Election of two representatives for the RESCUER supervisory board
17:30 Dinner at Hotel Terminus

Assignment for day 4.

Burgers' equation is given by,

$$\begin{aligned} u_t + \left(\frac{u^2}{2}\right)_x &= 0, \\ u(x,0) &= f(x), \\ [B_0 u]0,t) &= g_0(t), \\ [B_1 u](1,t) &= g_1(t) \end{aligned}$$

where B_0, B_1 are boundary operators, g_0, g_1 boundary data and f inital data.

- Determine well-posed boundary conditions that adapt to the flow through the boundaries.
- Using the previous codes as templates, implement the non-linear Burgers equation with the boundary conditions derived in the previous task. (The boundary conditions should be set, or not, depending on the local flow conditions.)
- Implement SBP artificial diffusion with variable diffusion coefficient.
- Let a wave enter the left boundary, travel across the domain and exit on the right. Try different diffusion coefficients (Roe, local and global Lax-Friedrichs).

RESCUER
WORKSHOP 1

Schedule for day 5 (06/12/2024):
9:00 Water Wave Dynamics (Henrik and Volker)
9:30 Challenges in Modeling prototypical model for nonlinear dynamics (Magnus)
10:30 Break, coffee
11:00 Coding
12:30 Lunch
13:30 Coding
14:30 Break
15:00 Coding
16:00 Break
16:15 Coding
17:00Closing Remarks and Workshop Summary